SYLLABUS

For

M.A./M.Sc. in MATHEMATICS

Four Semesters

(Effective from the academic session 2016 – 2017 and onwards)

DEPARTMENT OF MATHEMATICS

BANKURA UNIVERSITY

BANKURA

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Syllabus of M.Sc. Mathematics

[With effect from 2016-17]

The duration of M.Sc. course of studies in Mathematics shall be of two years consisting of four semesters each of six months duration leading to Semester-I, Semester-II, Semester-III and Semester-IV examinations at the end of each semester. Syllabus for M.Sc. courses in Mathematics is hereby framed according to the following schemes and structures.

The scheme of course will be Choice Based Credit System (CBCS) as per UGC guidelines. Total marks (total credits) for M.Sc. course is 1050 (84) with 250 marks (20 credits) in each of the 1st, 2nd and 4th semesters comprising of five papers each of 50 marks (4 credits) and 300 marks (24 credits) in the 3rd semester comprising of six papers each of 50 marks (4 credits). In each theoretical paper 20% marks is allotted for Internal Assessment. The subject teacher/(s) will evaluate the internal assessment. All the students will have to take the compulsory core papers, four major elective papers, one minor elective paper and the term paper, which are distributed over four semesters.

The "Foundation Courses" will run in Semester-I & in Semester-II. Students will select any one of the given papers as "Foundation Course" in each of those semesters. The foundation courses are to be conducted by the other Departments of the University. The courses shall have internal assessment only, and so, credit earned from these courses shall not be considered while preparing the final result. However, the candidates are required to be successful to obtain "**Satisfactory"** (as against "**Not Satisfactory")** grade to become eligible for the fourth semester examination/award of the M.Sc. degree.

The major elective courses that will run in a particular year in Semester-III & Semester-IV will be decided by the Department. The students have to give options for taking two major electives in each of the last two semesters from the clusters of elective papers offered in those semesters. Students have to take same title (if arises) of major elective courses both in Semester-III and Semester-IV. The option norm for selection of major elective courses is to be framed by the Department in each year based on the SGPA/CGPA available from the previous semester(s) of the students. However, the distribution of students to the major elective courses will be equally divided as far as practicable. All faculty members will supervise the students for term paper. Students will be almost equally distributed among the supervisors for the term paper. Term paper will be done from any topic on Mathematics and its Applications. The marks distribution of the Term paper is 25 Marks for the dissertation, 15 Marks for Seminar presentation and 10 Marks for Viva-Voce. The supervisor and external expert together will evaluate the term paper.

Course Structure

Sem. & Dur.	Course Type	Course Code	Name of the Course	Class hrs/week	L	T	P	Credit	I.A.	ESE
First Semester& 6 months	Core	Math-101C	Abstract Algebra	4	3	1	0	4	10	40
		Math-102C	Linear Algebra & Calculus of several variables	4	3	1	0	4	10	40
		Math-103C	Topology	4	3	1	0	4	10	40
		Math-104C	Techniques of Applied Mathematics (Generalized Functions, Special functions, Integral Equations)	4	3	1	0	4	10	40
		Math-105C(IA)	Internal Assignment (Numerical Analysis and NA-Practical using C-Prog.)	6	1	1	4	4	10	40
	Comp. Found.	Math-106CF	Communicative Skill and personality development	1	1	0	0	1	50	
Second Semester& 6 months	Core	Math-201C	Complex Analysis	4	3	1	0	4	10	40
		Math-202C	Functional Analysis	4	3	1	0	4	10	40
		Math-203C	Differential Geometry of curves and surfaces	4	3	1	0	4	10	40
		Math-204C	Ordinary differential equations and Partial differential equations	4	3	1	0	4	10	40
		Math-205C(IA)	Internal Assignment (Integral Transforms & Computational methods for PDEs)	6	2	0	4	4	10	40
	Elect. Found.	Math-206EF	Yoga and Life Skills Education/ Value Education and Human Rights	1	1	0	0	1	50	

Sem. & Dur.	Course Type	Course Code	Name of the Course		Class hrs/week	L	T	P	Credit	I.A.	ESE
Third Semester& 6 months	Core	Math-301C	Real Analysis		4	3	1	0	4	10	40
		Math-302C	Classical Mechanics		4	3	1	0	4	10	40
		Math-303C	Continuum Mechanics		4	3	1	0	4	10	40
	Major Elective -1 & Major Elective -2	Math-304ME & Math-305ME	Pure Group	Advanced Differential Geometry - I	4	3	1	0	4	10	40
				Operator Theory and Applications - I	4	3	1	0	4	10	40
				Geometric Mechanics on Riemannian manifolds - I	4	3	1	0	4	10	40
			Applied Group	Quantum Mechanics - I	4	3	1	0	4	10	40
				Elasticity – I	4	3	1	0	4	10	40
				Basics of Mathematical Modelling	5	2	1	2	4	20	30
				Space science - I	4	3	1	0	4	10	40
				Probability & Statistics	5	2	1	2	4	20	30
				Dynamical Systems	4	3	1	0	4	10	40
				Variational Principles	4	3	1	0	4	10	40
	Minor Elective (Open Elective)	Math-306ME(ID)	Computer Applications (Latex, MatLab, Mathematica)		6	1	1	4	4	10	40

Sem. & Dur.	Course Type	Course Code	Name of the Course		Class hrs/week	L	T	P	Credit	I.A.	ESE
Fourth Semester& 6 months	Core	Math-401C	Operations Research / Set Theory and Mathematical Logic		4	3	1	0	4	10	40
		Math-402C	Graph Theory & Field Theory		4	3	1	0	4	10	40
	Major Elective-3 & Major Elective-4	Math-403ME & Math-404ME	Pure Group	Advanced Differential Geometry – II	4	3	1	0	4	10	40
				Operator Theory and Applications - II	4	3	1	0	4	10	40
				Geometric Mechanics onRiemannian manifolds - II	4	3	1	0	4	10	40
			Applied Group	Quantum Mechanics - II	4	3	1	0	4	10	40
				Elasticity – II	4	3	1	0	4	10	40
				Space science - II	4	3	1	0	4	10	40
				Introduction to PDE-constrained optimization	4	3	1	0	4	10	40
				Modelling and Analysis of Biological systems	4	3	1	0	4	10	40
				Computational Fluid Dynamics	4	3	1	0	4	10	40
	Term Paper	Math-405T(IA)	Internal Assignment (Mathematics and its Applications)		6	0	2	2	4	10	40

Meaning of the symbols: ***L = Lecture, T = Tutorial, P = Practical, I.A. = Internal Assessment, ESE= End Semester Examination***

SEMESTER-I

Paper: Math-101C

Abstract Algebra

Total Lectures: 50 ***(Marks – 50)***

Groups: Homomorphism, Isomorphism and Automorphism of groups, Normal Subgroups, Quotient Groups, Isomorphism and Correspondence Theorems,Cayley's Theorem, Groups of order 4 and 6, D_4 and $Q_{8,}$ Simple groups. (10L)
Group action on a set, Class equation, p-groups, Cauchy's theorem, Converse of Lagrange's theorem for finite Abelian groups, Sylow theorems and some of its applications, Internal and external direct product. (10L)
Normal and Subnormal Series, Composition Series, Solvable Groups and Nilpotent Groups, Jordan-Hölder Theorem and its applications,
finitely generated Abelian groups, free Abelian groups. (5L)

Rings: Ideals and Homorphisms, Prime and Maximal Ideals, Quotient Field of an Integral Domain. (6L)
Polynomial Rings, Divisibility Theory: Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss' Theorem. (11L)
Noetherian and Artinian Rings, Hilbert Basis Theorem, Cohen's Theorem. (3L)

Modules: Left and Right Modules over a ring with identity, Cyclic Modules, Free Modules, Artinian and Notherian modules Fundamental Structure Theorem for finitely generated modules over a PID and its applications to finitely generated abelian groups. (5L)

References:

1. Dummit, Foote – *Abstract Algebra,* Wiley
2. Malik, Mordeson&Sen – *Fundamentals of Abstract Algebra,* McGraw-Hill
3. I. N. Herstein – *Topics in Algebra*, Wiley
4. P. B. Bhattacharya, S. K. Jain & S. R. Noyapal – *Basic Abstract Algebra*, Cambridge
5. Hungerford – *Algebra*, Springer

Paper: Math-102C

Group A

Linear Algebra

Total Lectures: 30 ***(Marks – 30)***

Vector Spaces, Euclidean spaces, Linear transformation in finite dimensional spaces and its matrix representation,rank and nullity, Linear functional, Dual space and Dual basis, Double Dual, Transpose of a linear transformation and matrix representation of the transpose of a linear transformation, Eigen values and Eigen vectors, Characteristic polynomials, Cayley-Hamilton theorem, Minimal polyniomial, Invariant subspaces, Diagonalizability and Triangulability, Direct sum Decompositions, Projection of Linear transformations, Invariant Direct sums, Primary Decomposition Theorem.

Jordon Canonical Form, Rational Canonical Form, Computation of Invariant factors and Elementary divisors.

Inner Product spaces: Real and Complex Inner Product Spaces, Orthonormal Basis, Gram-Schmidt Orthogonalization Process, Linear Functionals on an Inner Product Space and its relation with the inner product, Orthogonal Projection.

Bilinear Form and its Matrix Representation, Symmetric Bilinear Form, Quadratic Form, Classification of Quadric Forms, Sylvester's law of inertia.

References:

1. K. Hoffman & Kunze – *Linear Algebra* (Prentice – Hall)
2. Ramchandra Rao & P. Bhimasankar – *Linear Algebra* (TMH)
3. V. A. Iyin&Poznyak – *Linear Algebra* (Mir)
4. S. Lang – *Linear Algebra* (Springer Verlag)
5. Helmos – *Finite dimensional vector spaces*
6. Schaum's series – *Linear Algebra*
7. M. Postnikov – *Lectures in Geometry, Linear Algebra and Differential Geometry*

Group B

Calculus of Several Variables

Total Lectures: 20 ***(Marks – 20)***

Continuity and derivative of functions of several variables, partial derivatives, directional derivatives, smooth functions, real analytic functions, diffeomorphisms, Taylors theorem, chain rule, differential forms, exterior product, inverse function theorem (local and global), implicit function theorem. (20L)

Text Book:

1. M. Spivak–*Calculus on manifolds*, Addison-Wesley Pub. Comp.

Reference Books:

1. J. R. Munkres–*Analysis on manifolds*, Addison-Wesley Pub. Comp., 1991.
2. R. Courant and F. John –*Introduction to calculus and analysis, Volume-II*, Springer-Verlag, New York, 2004.

Paper: Math-103C

Topology

Total Lectures: 50 ***(Marks – 50)***

Topological spaces: Definition, Examples, operations (Union and Intersection) of topologies, Weak and Strong topologies, open sets, Neighbourhood System at a point, Base for a topology, Necessary and sufficient conditions for a base of a topology, Limit point and derived set of a set, closed sets and their algebra, closure of a set, Interior of a set, Kuratowski Closureoperator and resulting topology, Subspace and relative topology, Continuous functions, Open functions, closed functions, Homeomorphism, first and second countable spaces, Lindelöfspaces, separable spaces and their relationship.(20L)
Separation axioms: T_0, T_1, T_2 Spaces, Regular spaces, T_3Space, Completely regular spaces, Normal space, $T_{3½}$ Space, T_4 space, their properties and their relationships, Urysohn's lemma and Tietze's extension theorem, Metrizable space and Uryshon's Metrization Theorem (Statement only)(10L).
Compactness: Open cover, Compact space, Compact sets , Finite intersection properties, Characterization of compact sets of reals with usual topology, Continuous image of compact spaces, locally compact space, Paracompact spaces (7L).
Connectedness: Connected spaces, Connected sets of reals with usual topology, Union of connected space,Continuous image of connected spaces, Components (7L).
Product Topology: Product spaces, projection mappings, quotient topology, quotient spaces, product of compact spaces, Tychonoff Theorem, Product of Connected spaces. (6L)

Text Book:

1. J. R. Munkres – *Topology, a first course*

References:

2. W. J. Thron – *Topological Structures*
3. K. D. Joshi – Introduction to *General Topology*
4. J. L. Kelly – *General Topology*
5. G. F. Simmons – *Introduction to Topology & Modern Analysis.*

Paper: Math-104C

Techniques of Applied Mathematics

Total Lectures: 50 ***(Marks – 50)***

Generalized Functions: Distributions,Generalized functions and its elementary properties; Addition, Multiplication, Transformation of variables. Generalized function as the limit of a sequence of good functions, Differentiation of generalized function. Simple examples, Dirac-Delta function,Plemelz' formula.
Antiderivative, Regularization of divergent integral: Simple examples (8L)
Special functions: Ordinary point and singularity of a second order linear differential equation in the complex plane; solution about an ordinary point, solution of Hermite equation, Hermite polynomial; Regular singularity,Fuch's theorem, solution about regular singularity with examples,Frobenius' method, Solution of Hypergeometric equation.(6L)
Legendre polynomial, its generating function; Rodrigue's formula, recurrence relations and orthogonality properties;Associated Legendre functions, Legendre functions of second kind,

expansion of a function in a series of Legendre Polynomials, spherical harmonics, Graph of Legendre function.(8L)
Bessel functions of first and second kind, its generating function, recurrence relations, Modified Bessel functions and their recurrence relations, orthonormality. Bessel series, Graph of Bessel function.(7L)
Solution of Legendre, Laugurre and Bessel equation. (4L)
Integral Equations: Origin and classification. Reduction of Initial value and boundary value problems to integral equations, Existence and Uniqueness of solutions of Fredholm and Volterra Integral equations; examples, Solution of Fredholm integral equation with degenerate kernel, symmetric kernel. Fredholm alternative, Numerical solution of Fredholm integral equations,Volterra integral equation of first and second kind. Resolvent kernel, Neumann series, Solution by Method of successive approximations, Difference kernel, Laplace Transform method, Examples, Numerical solution of Volterra integral equations, Singular integral equation, Solution of Abel's integral equation. (17L)

References:

1. Gelfand&Shilov – *Generalised Functions* (Academic Press)
2. E. D. Rainville – *Special Functions* (Macmillan)
3. I. N. Sneddon–*Special Functions of mathematical Physics & Chemistry* (Oliver & Boyd, London)
4. N. N. Lebedev–*Special Functions and Their Applications* (PH)
5. S. G. Mikhlin – *Integral Equation* (Pergamon Press)
6. F. G. Tricomi – *Integral Equation* (Interscience Publishers)
7. WE. V. Lovit. – *Linear Integral Equations* (Dover Publishers)

Paper: Math-105C(IA)

Group A
Numerical Analysis

Total Lectures: 20 ***(Marks – 20)***

Solving system of linear equations: Existence & uniqueness of solutions. *Iterative Methods*: Jacobi, Gauss-Siedl and SOR methods. Operational count: operational counts of exact methods and iterative methods (3L), Multigrid Methods (3L).
System of Non-Linear Equations: Fixed point theory, Conditions of convergence, Newton's Method (4L), Variations of Newton's method. (3L)
Eigen Value Problems: Power method (3L)
ODE: IVP: Concept of singlestep and multistep methods; Euler and Runge-Kutta methods, Generating solution curve of IVP using Runge-Kutta methods, Milne's method, Adam-Basforthmethod. (4L)

References:

1. F. B. Hildebrand – *Introduction to Numerical Analysis*
2. Demidovitch and Maron – *Computational Mathematics*
3. Jain, Iyengar and Jain – *Numerical Methods for Scientific and Engg. Computation*
4. A. Gupta and S. C. Basu – *Numerical Analysis*
5. Scarborough – *Numerical Analysis*
6. Atkinson – *Numerical Analysis*
7. Raulstan – *Numerical Analysis.*

Group B

Numerical Analysis Practical using C-Programming

Total Lectures: 30 ***(Marks – 30)***

Practical: Using C programming

Addition, power, GCD, finding maximum among some numbers, finding prime numbers, generating Fibonacci Series, Matrix addition, Matrix multiplication. (5L)
Converting a Matrix into its row reduced echelon form/Lower or Upper triangular form if nonsingular square matrix, Calculation of Determinant of a Square matrix, Detection whether it is singular or not, Gauss Jordan Elimination. (7L)
Solving a system of equations with the coefficient matrix of the form of lower triangular matrix/ Upper triangular matrix; LU decomposition, Solution using LU decomposition, Gauss Elimination. (7L)
Gauss Jacobi iteration, Gauss-Siedel Iteration, Largest Eigen Value by Power method. (6L)
Using Runge-Kutta method for solving a differential equation with proper initial condition and plotting the solution. (5L)

References:

1. Xavier, C. – *C Language and Numerical Methods* (New Age International (P) Ltd. Pub.)
2. F. Scheid – *Computers and Programming* (Schaum's series)
3. *Gottfried, B. S. – Programming with C* (TMH)
4. Balaguruswamy, E. – *Programming in ANSI C* (TMH).

SEMESTER-II

Paper: Math-201C

Complex Analysis

Total Lectures: 50 ***(Marks – 50)***

Complex integration, line integral and its basic properties, index of a curve, winding number, connectedness of the complex plane, locally constant and globally constant functions, pathhomotopy, Cauchy's fundamental theorem (homotopy version), Cauchy's integral formula and higher derivatives, power series expansion of analytic functions. (14L)
Zeros of analytic functions and their limit points, identity theorem, entire functions, Liouville's theorem,fundamental theorem of algebra. (6L)
Simply connected region and primitives of analytic functions, Morera's theorem,open mapping theorem. (5L)
Singularities, Laurent's series expansion and classification of singularities and Casorati-Weierstrass'stheorem,Cauchy's residue theorem and evaluation of improper integrals. (10L)
Argument principle, Rouche's theorem andits application,maximum modulus theorem. (5L)
Harmonic conjugates, conformal mappings, Schwarz's Lemma and its applications,analytic continuation. (10L)

Text Book:

1. J. B. Conway – *Functions of one Complex Variable*(Narosa Publishing House)

References:

2. R. B. Ash – *Complex Variable* (A.P.)
3. Punoswamy – *Functions of Complex Variable*
4. Titchmarsh – *Theory of Functions.*
5. Churchil, Brown – *Complex Variable* (MH)

Paper: Math-202C

Functional Analysis

Total Lectures: 50 ***(Marks – 50)***

Normed linear spaces, continuity of norm function, Banach spaces, Spaces $\mathbf{C}^n$, C[a,b] with supmetric, c_0, l_p ($1 \le p \le\infty$) and other spaces, linear operators, boundedness and continuity, bounded and unbounded linear operators, Baire category theorem. Banach contraction principle – application to Picard's existence theorem and Implicit function theorem, Hilbert spaces with various examples such as l_2 spaces, $L_p[a, b]$ ($1 \le p \le\infty$) etc; C-S inequality, Parallelogram law, Pythagorean law, Minkowski inequality, completion of metric space.,equicontinuous family of functions, compactness in C[0,1] (Arzela-Ascoli's Theorem), convex sets in linear spaces, properties of normed linear spaces, finite dimensional normed linear spaces, Riesz's Lemma, and its application in Banach spaces, convergence in Banach Spaces, equivalent norms and their properties, principle of uniform boundedness (Banach-Steinhaus), open mapping theorem, closed graph theorem, extension of continuous linear mapping, invertible mappings and their properties, linear functional, Hahn-Banach theorem and its applications, conjugate spaces, reflexive spaces, properties of strong and weak convergence, adjoint (conjugate) operators and their properties, continuity of inner product, convergence, orthogonality and orthogonal decomposition of a Hilbert Space, complete orthonormal set, Bessel's inequality, Perseval's equality, minimization of norm problem, complete orthonormal set, Riesz-Fischer's theorem, Riesz representation theorem for bounded linear functional over a Hilbert space.

References:

1. Lusternik&Sobolov – *Elements of Functional Analysis*
2. B. K. Lahiri – *Elements of Functional Analysis*
3. Bachman and Narici – *Functional Analysis*
4. Brown & Page – *Functional Analysis*
5. W. Rudin – *Functional Analysis*

Paper: Math-203C

Differential Geometry of Curves and Surfaces

Total Lectures: 50 ***(Marks --50)***

Geometry of Curves:
Definition of curves in R^n with examples, arc-length, reparametrization, level curves and parametric curves, curvature of plane curves and space curves, properties of plane curves, torsion of space curves, basic properties of plane and space curves, Serret-Frenet formulae. (10L)

Geometry of Surfaces:
Definition of surfaces with various examples, smooth surfaces with examples,tangent, normal and orientability of surfaces,quadric surfaces. (10L)
First fundamental form, length of curves on surfaces, isometries of surfaces, conformal mapping of surfaces, surface area. (10L)
Curvature of surfaces, second fundamental form, curvature of curves on surfaces, normal and principal curvatures, Meisuner's theorem, Euler's theorem. Gaussian and mean curvature, pseudosphere, flat surfaces, surfaces of constant mean curvature. (10L)
Geodesics, Geodesic equations, Geodesic on surfaces of revolution, Geodesic as shortest path, Geodesic coordinates,minimal surfaces with examples, Gauss Theorem Egregium and its applications, developable surfaces. CodazziMainardi equation, third fundamental form, compact surfaces of constant Gaussian curvatures, Gauss-Bonnet theorem for simple closed curves (Statement only). (10L)

Text Book:

1. Andrew Pressley– Elementary Differential Geometry, Springer-Verlag, 2001, London (Indian Reprint 2004).

Reference Books:

1. Manfredo P. Do Carmo– Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood, Cliffs, New Jersey, 1976.
2. Barrett O'Neill– Elementary Differential Geometry, 2nd Ed., Academic Press Inc., 2006.

Paper: Math-204C

Ordinary Differential Equations and Partial Differential Equations

Total Lectures: 50 ***(Marks – 50)***

ODE: First order system of equations: Well-posed problems, simple illustrations. Peano's and Picard's theorems (statements only).(2L)
Linear systems, non-linear autonomous system, phase plane, critical points, stability, Liapunov stability, undamped pendulum.(5L)
Linear ordinary differential equations, generalized solution, fundamental solution, inverse of a differential operator.
Nonhomogeneous ordinary differential equations, Two-point boundary value problem for a second-order linear nonhomogeneous O.D.E.Self adjoint operator. Sturm-Liouville problem.Costruction of Green's functions with examples. Orthogonal property of solutions of Sturm-Liouville problem.
Analogy between linear simultaneous algebraic equations and linear differential equation. (3L)

Partial Differential Equation: Partial Differential Equation of the first order.Cauchy's problem for the first order equations.Linear first order equations.Lagrange's equation. General solution. Integrals passing through a given curve. Orthogonal surfaces. Nonlinear PDE of the first order. Cauchy's method of Characteristics.Charpit's method. Examples.(7L)
Second order linear P.D.E: Classification, reduction to normal form; characteristic curves. Solution of linear hyperbolic equations byRiemann method. Riemann-Green's function. (7L)
Laplace equation: Occurrence of Laplace equations.Boundary value problems: Dirichlet (interior and exterior) and Neumann (interior and exterior). Separation of variables.Green's function for Laplace's equation. Its properties and methods of construction. (10L)
Wave equation: Occurrence of wave equation. D'Alemberts solution.Riemann-Volterra solution of one dimensional wave equation. Domain of influence and domain of dependence.Solution by separation of variables method.Solution by method of integral transforms.Vibrating membranes. (8L)
Diffusion equation: Occurrence of diffusion equation. Elementary solutions.Solution by integral transform technique.Separation of variables for rectangular and circular plate problems. Green's function .Bilinear expansion for Green's function.Finding the elementary solution of diffusion equation. (8L)

References:

1. I. N. Sneddon – *Integral Transforms* (MacGraw-Hill)
2. T.Amarnath – *Partial Differential Equation*
3. I. N. Sneddon – *Partial Differential Equation*
4. P. Phoolan Prasad & R. Ravichandan – *Partial Differential Equations*
5. F. John – *Partial Differential Equations*
6. Williams - *Partial Differential Equations*
7. Epstein - *Partial Differential Equations*
8. Chester - *Partial Differential Equations*.

Paper: Math-205C(IA)

Internal Assignment

(Integral Transforms & Computational Methods for PDEs)

Total Lectures: 50 ***(Marks – 50)***

Group – A

Integral Transforms

Total Lectures: 20 ***(Marks --20)***

Integral Transforms: Fourier Transform and its properties, Inversion formula of F.T.; Convolution Theorem; Parseval's relation. Finite Fourier transform and its inversion formula. Applications. (8L)
Laplace's Transform and its properties. Inversion by analytic method and by Bromwitch path. Lerch's Theorem. Convolution Theorem; Applications. Fourier and Laplace transform of generalized function.Applications, Hankel Transformations (12L)

Group – B

Computational Methods for PDEs

Total Lectures: 30 ***(Marks -- 30)***

Discretization Methods: Finite Difference Method for Partial Differential equations; Consistency, stability and convergence; Applications to Elliptic equations, Forward Difference Explicit Methods, Truncation errors for forward Difference Explicit Methods, Crank Nicolson Implicit method, Truncation errors for implicit methods; Applications to parabolic and hyperbolic equations.

References:

1. Analysis of Discretization Methods for Ordinary Differential Equations – Hans J. Stetter
2. Computational Fluid Dynamics: C. A. J. Fletcher (Springer)
3. Introduction to Computational Fluid Dynamics: P. Niyogi, S. K. Chakraborty, M. Laha.

SEMESTER-III

Paper Math-301C

Real Analysis

Total Lectures: 50 ***(Marks – 50)***

Monotone functions and their discontinuities, Functions of bounded variation along with their properties, Riemann-Stieltjes integral, its existence and convergence problem. (15L)

Lebesgue outer measure, measurable sets and their properties, Lebesgue measure, measurable function, simple function and measurable function, Lebesgue integral of a non-negative measurable function using simple functions, Lebesgue integral of functions of arbitrary sign, basic properties, monotone convergence theorem and its consequences, Fatou's lemma, Lebesgue dominated convergence theorem, comparison of Lebesgue integral and Riemann integral. (25L)

Fourier series, Dirichlet's integral, Riemann-Lebesgue theorem, pointwise convergence of Fourier series of functions of bounded variation. (10L)

References:

1. I. P. Natanson – *Theory of Functions of a Real Variable,* Vol. I
2. C. Goffman – *Real Functions*
3. Burkil&Burkil – *Theory of Function of a Real Variable*
4. Goldberg – *Real Analysis*
5. Royden – Real Analysis
6. Limaye – *Functional Analysis*
7. Vulikh – *Function of a Real Variable*
8. Lahiri& Roy – *Theory of Functions of a Real Variable*
9. Apostol – *Real Analysis*

10. Shah &Saxena – *Functions of a Real Variable*
11. Charl'sScwarz – *Measure, Integration and Function space*
12. W. Rudin - *Real and complex analysis.*

Paper: Math-302C

Classical Mechanics

Total Lectures: 50 ***(Marks – 50)***

Constraints, Basic problems with constraint forces,Principle of Virtual Work,D Alembert's principle, Work energy relation for constraint forces of sliding friction.
Lagrangian formulation of Dynamics: Degrees of freedom, Generalized coordinates, Lagrange's equations of motion for holonomic and non-holonomic systems, Kinetic energy function, Theorem on total energy, linear generalized potentials, generalized momenta and energy. (5L)
Ignoration of co-ordinates, Routh'sprocesss for the ignoration of co-ordinates, Rayleigh's dissipation function. (6L)
Calculus of variation:Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema, Variational methods for boundary value problems in ordinary and partial differential equations, Brachistochrone problem. (6L)
Invariance of Euler-Lagrange equation of motion under generalized coordinate transformation. Configuration space and system point, Hamilton's principle, Hamilton's canonical equations of motion. Principle of energy, Principle of least action. (6L)
Generating Function, Canonical Transformations, Poisson Bracket. (4L)
Theory of small oscillations, Normal co-ordinates, Euler's dynamical equations of motion of a rigid body about a fixed point, Torque free motion, Motion of a top on a perfectly rough floor, Stability of top motion, Motion of a particle relative to rotating earth, Foucault's pendulum.(8L)
Special Theory of Relativity: Postulates of Special Relativity, Lorentz Transformation, Consequences of Lorentz Transformation, Lorentz group, Length contraction, time dilation, transformation of velocity, law of composition of velocity, mass variation, transformation of momentum, Energy mass relation$\boldsymbol{E = mc^2}$.(15L)

Reference:

1. F. Chorlton – *A Text Book of Dynamic*
2. Synge and Griffith – *Principles of Mechanics*
3. D. T. Green Wood – *Classical Dynamics*
4. E. T. Whittaker – *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*
5. K. C. Gupta – *Classical Mechanics of Particles and Rigid Bodies*
6. I. S. Sokolnikoff – Mathematical Theory of Elasticity
7. T. J. Chung – *Continuum Mechanics* (Prentice – Hall)
8. Truesdell – *Continuum Mechanics* (Schaum Series)
9. Molla – *Theory of Elasticity*
10. F. Gantmacher – *Lectures in Analytical Mechanics*
11. J. L. Bansal – *Viscous Fluid Dynamics* (Oxford)
12. H. Goldstein – *Classical Mechanics*
13. R. N. Chatterjee – *Contunuum Mechanics*
14. N.C. Rana and P.S.Joag—Classical Mechanics

Paper: Math-303C

Continuum Mechanics

Total Lectures: 50 ***(Marks – 50)***

Continuous media: Deformation. Lagrangian and Eulerian coordinates. Relationship between Lagrangian and Eulerian Coordinates. Conservation of mass. Strain tensor. Rate of deformation tensor. Co-ordinate transformation of strains. Principal strains. Principal strain invariants. Examples. Maximum shear strains. Mohr circle representation. Compatibility equations.(12L)
Equilibrium and kinetics: Forces and stresses. Basic balance laws: Balance of linear and angular momentum; Cauchy's first and second laws of motion. Coordinate transformation of stresses. Principal stresses. Principal stress invariants. Examples. Mohr circle representation, the deviatoric stress tensor.(12L)
Constitutive equations for linear elastic solids. Generalized Hook's law. Monotropic, orthotropic, Transeversely isotropic and isotropic material. Lame constants. Navier equations. (6L)
Fluid flow problems: Definition of a fluid, Fluid properties, Classification of flow phenomena, Equations of fluid motion, Navier-Stokes equations (compressible and incompressible), Euler's equations (compressible and incompressible), Non-dimentionalization of equations, Reynolds number and Prandtl number. RANS equations. Introduction to turbulence and its modelling, introduction to DNS, LES and RANS simulations. (20L)

References:

1. T. J. Chung – *Applied Continuum Mechanics* (Prentice – Hall)
2. C. Truesdell – *Continuum Mechanics* (Schaum Series)
3. I. S. Sokolnikoff – *The Mathematical Theory of Elasticity (McGraw Hill)*
4. Molla – *Theory of Elasticity*
5. F. Gantmacher – *Lectures in Analytical Mechanics*
6. J. L. Bansal – *Viscous Fluid Dynamics* (Oxford)
7. H. Lamb – *A Treatise on the Mathematical Theory of the Motion of Fluids (Cambridge University Press)*
8. R. N. Chatterjee – *Contunuum Mechanics*
9. A. J. Chorin and J. E. Marsden – *Mathematical Introduction of Fluid Mechanics* (Springer)
10. G. K. Batchelor – *Fluid Dynamics* (Cambridge University Press)

Papers: Math-304ME & Math-305ME

Advanced Differential Geometry-I

Total Lectures: 50 ***(Marks- 50)***

Definition and various examples of differentiable manifolds, examples of non-Hausdorff, non-connected and non-2nd countable manifolds, topology of manifolds, tangent spaces,cotangent spaces, Jacobian map, vector fields, integral curves, one parameter group of transformations, Lie derivatives, immersions and embeddings, distributions. (30L)

Multilinear maps, tensors, tensor products, tensor fields, exterior algebra, exterior derivatives. (10L)
Topological groups, Lie groups and Lie algebras, Lie subgroups, Heisenberg groups, product of two Lie groups, one parameter subgroups and exponential maps, examples of Lie groups, homomorphism and isomorphism, Lie transformation groups, general linear groups. (10L)

References:

1. B. B. Sinha, *An Introduction to Modern Differential Geometry*, Kalyani Publishers, New Delhi, 1982.
2. K. Yano and M. Kon, *Structure of Manifolds,* World Scientific Publishing Co. Pvt. Ltd., 1984.
3. John M. Lee, *Introduction to Smooth Manifolds*, 2nd Ed., Springer-Verlag, 2012.
4. William H. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Academic Press, New York, 1975.
5. S. Lang, *Introduction to Differential Manifolds*, John Wiley and Sons, New York, 1962.
6. S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry, Vol. 1,*Interscience Press, New York, 1969.

Operator Theory and Applications-I

Total Lectures: 50 ***(Marks- 50)***

Adjoint operators over normed linear spaces, their algebraic properties, compact operators on normal linear spaces, sequence of compact operators, compact extensions, weakly compact-operators. (10L)
Operator equation involving compact operators, Fredhölm alternative, adjoint operators on Hilbert-spaces, self-adjoint operators and their algebraic properties; unitary operators, normal operators in Hilbert spaces, positive operators, their-sum, product; monotone sequence of positive operators, square-root of positive operator, projection operators. (15L)
Their sum and product, idempotent operators, positivity norms of projection operators; limit of monotone increasing sequence of projection operators. (25L)

References:

1. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966
2. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd
3. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
4. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994
5. G.F. Simmons- *Introduction to topology and Modern Analysis* ,McGraw Hill, New York, 1963
6. N. Dunford and J.T. *Schwartz-Linear Operators, Vol-1&II*, Interscience, New York,1958
7. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3rdEdn., 1990
8. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970
9. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958
10. L.V. Kantorvich and G.P. akilov-*Functional Analysis*, Pergamon Press,1982
11. Vulikh- Functional Analysis
12. J. Tinsley Oden&Leszek F. Dernkowicz- *Functional Analysis*, CRC Press Inc, 1996.
13. Lipschitz-General Topology, Schaum Series.

Basics of Mathematical Modelling

Total Lectures: 50 ***(Marks- 50)***

Introduction: Description, objectives, classification and stages of mathematical modelling. (10L)

Model Building: Understanding the system to be modelled and its environment, system analysis, formulating the mathematical equations, solving equations efficiently. (10L)

Studying models: Dimensionless form, asymptotic behavior, sensitivity analysis, interpretation of model outputs. (10L)
Testing models: Verification of assumptions and model structures, prediction of unused data, estimating model parameters, comparing alternative models. (10L)
Using models: Predictions with estimates of precision, decision support. Discussion and practical applications. (10L)

References:

1. Michael D. Alder –*An Introduction to Mathematical Modelling* (HeavenForBooks.com)
2. AlfioQuarteroni–*Mathematical Models in Science and Engineering*, Notices of the AMS, Vol. 56(1), 2009.
3. Clive L. Dym– *Principles of Mathematical Modeling (2nd Edition)*, (Academic Press), 2004.
4. R. Illner, C. Sean Bohun, S. McCollum, T. van Roode–*Mathematical Modelling.*

Variational principles

Total Lectures: 50 ***(Marks- 50)***

Variational problems with fixed boundaries. Variational problems with moving boundaries. Euler-Lagrange equation. Necessary and sufficient conditions for an extremum. Variational problems with subsidiary conditions. Eigenvalue problems. Direct method in Variational problems. Variational formulation for linear and nonlinear problems. Variational problems in fluid flow and heat transfer.

References:

1. A. S. Gupta, PHI Learning Pvt. Ltd.– *Calculus of Variations with Applications*, 2011.
2. M. Bendersky– *The Calculus of Variations*.

Geometric Mechanics on Riemannian manifolds-I

Total Lectures: 50 ***(Marks- 50)***

Differentiable manifolds: Embedded manifolds in R^N, The tangent space, the derivative of a differentiable function, Tangent and cotangent bundles of a manifold, Discontinuous action of a group on a manifold, Immersions and embeddings. Submanifolds, Partition of unity. Vector fields, differential forms and tensor fields, Lie derivative of tensor fields, The Henri Cartan formula. Pseudo-Riemannian manifolds: Affine connections, The Levi-Civita connection, Tubular neighborhood, Curvature, E. Cartan structural equations of a connection. (10L)

Newtonian mechanics: Galilean space-time structure and Newton equations, Critical remarks on Newtonian mechanics. Mechanical systems on Riemannian manifolds: The generalized Newton law, The Jacobi Riemannian metric, Mechanical systems as second order vector fields, Mechanical systems with holonomic constraints, Some classical examples, The dynamics of rigid bodies, Dynamics of pseudo-rigid bodies, Dissipative mechanical systems. Mechanical systems with non-holonomic constraints: D'Alembert principle, Orientability of a distribution and conservation of volume, Semi-holonomic constraints, the attractor of a dissipative system. Hyperbolicity and Anosov systems. Vakonomic mechanics: Hyperbolic and partially hyperbolic structures, Vakonomic mechanics, Some Hilbert manifolds, Lagrangianfunctionals and *D*-spaces, D'Alembert versus vakonomics. (10L)

Special relativity: Lorentz manifolds, the quadratic map of R^{n+1}, Time-cones and time-orientability of a Lorentz manifold, Lorentz geometry notions in special relativity, Minkowski space-time geometry, Lorentz and Poincar´e groups. (10L)

General relativity: Einstein equation, Geometric aspects of the Einstein equation, Schwarzschild space-time, Schwarzschild horizon, Light rays, Fermat principle and the deflection of light. Hamiltonian and Lagrangian formalisms: Hamiltonian systems, Euler–Lagrange equations. (10L)

Quasi-Maxwell form of Einstein's equation: Stationary regions, space manifold and global time, Connection forms and equations of motion, Stationary Maxwell equations, Curvature forms and Ricci tensor, Quasi-Maxwell equations. (10L)

References:

1. Ovidiu Calin, Der-Chen Chang– *Geometric mechanics on Riemannian manifolds*, Springer-Verlag, 2006.

Probability & Statistics

Total Lectures: 50 ***(Marks – 50)***

Classes of sets: Sequence of sets, limsup, liminf, limit of sequence of sets field, σ field, σ field generated by a class, Borel σ field. (4L)
Non parametric estimation: Estimability, Kerner U Statistics, U Statistics theorem for one sample and two samples. (2L)
Probability measure, Probability space, properties of probability measure – continuity mixture of probability measures, Lebesque&Lebesque-Stelties measures. Measures on R, independence of events. (4L)
Problem of testing hypothesis, Neumann Pearson lemma, monotone likelihood ratio property, UNP tests, UNPU test. (4L)
Regression analysis: Simple linear regression, Multiple linear regression, Logistic regression. (8L)
Multivariate data analysis: Overview of multivariate methods, Principal components analysis, Partial least squares regression, Cluster analysis, Discriminant function analysis, Canonical variate analysis. (10L)
Time series modelling: Stationary time series models, Trend models, Seasonal Models, Models with explanatory variables. (8L)
Practical on data analysis using statistical software. (10L)

References:

1. *An Introduction to Probability and Statistics* by V.K. Rohatgi& A.K. Md. E. Saleh.

2. *Introduction to Probability and Statistics* by J.S. Milton & J.C. Arnold
3. *Introduction to Probability Theory and Statistical Inference* by H.J. Larson
4. *Introduction to Probability and Statistics for Engineers and Scientists* by S.M. Ross
5. *A First Course in Probability* by S.M. Ross
6. *Probability and Statistics in Engineering* by W.W. Hines, D.C. Montgomery, D.M. Goldsman& C.M. Borror
7. *Lectures in Probability* by M. Kac (for example on independent events)
8. C.K. Wong (1972) – *A note on mutually independent events*. Annals of Statistics, V. 26, 27(for example on independent events).

Elasticity-I

Total Lectures: 50 ***(Marks- 50)***

Generalised Hooke's law Orthotropic and transversely isotropic media. Stress-strain relations in isotropic elastic solid. (5L)
Saint-Venant's semi-inverse method of solution (Statement), formulation of torsion problem and the equations satisfied by the torsion function and the boundary condition. Formulation of torsion problem as an internal Neumann problem,Dirichlet's problem and Poisson's problem, Prandtl's stress function,shearing stress in torsion problem, Solution of torsion problem for simple sections Method of sol. of torsion problem by conformal mapping. (20L)
Flexure problem: Reduction of flexure problem to Neumann problem, solution of flexure problem for simple sections. (8L)
Potential energy of deformation. Reciprocal theorem of Betti and Rayleigh, theorem of min. Potential energy. (8L)
Plane problem: plane strain, plane stress, generalised plane stress. Basic equations. Airy's stress function. Solution in terms of complex analytic function. (9L)

References:

1. Y. A. Amenzade – *Theory of Elasticity* (MIR Pub.)
2. A. E. H. Love – *A treatise on the Mathematical Theory of Elasticity, CUP, 1963.*
3. I. S. Sokolnikoff– *MathematicalTheory of Elasticity, Tata McGraw Hill Co., 1977.*
4. W. Nowacki – *Thermoelasticity* (Addison Wesley)
5. Y. C. Fung- *Foundations of Solid Mechanics*, PHI, 1965.
6. S. Timoshenk and N. Goodies, *Theory of Elasticit*y, McGrwa Hill Co., 1970.
7. N. I. Muskhelishvili- *Some Basic Problems of the mathematical theory of Elasticity*, P. Noordhoff Ltd., 1963.

Dynamical Systems

Total Lectures: 50 ***(Marks – 50)***

Linear systems:
Linear autonomous systems, existence, uniqueness and continuity of solutions, diagonalization of linear systems, fundamental theorem of linear systems, the phase paths of linear autonomous plane systems, complex eigen values, multiple eigen values, similarity of matrices and Jordon canonical form, stability theorem, reduction of higher order ODE systems to first order ODE systems, linear systems with periodic coefficients.

Nonlinear systems:
The flow defined by a differential equation, linearization of dynamical systems (two, three and higher dimension), Fixed Points, Stability: (i) asymptotic stability (Hartman's theorem), (ii) global stability (Liapunov's second method).
Periodic Solutions (Plane autonomous systems):
Translation property, limit set, attractors, periodic orbits, limit cycles and separatrix, Bendixon criterion, Dulac criterion, Poincare-Bendixon Theorem, index of a point, index at infinity.

Bifurcations:
Saddle-node, transcritical and pitchfork bifurcations, hopf- bifurcation.

Linear difference equations:
Difference equations, existence and uniqueness of solutions, linear difference equations with constant coefficients, systems of linear difference equations, qualitative behavior of solutions to linear difference equations.

Nonlinear difference equations (Map):
Steady states and their stability, the logistic difference equation, systems of nonlinear difference equations, stability criteria for second order equations, stability criteria for higher order system.

References:

1. Beltrami, E., *Mathematics for Dynamic Modeling,* Academic Press, Orlando, Florida, 1987.
2. Kapur, J. N., *Insight into Mathematical Modeling,* Indian National Science Academy, New Delhi, 1983.
3. Meyer, W., *Concepts of Mathematical Modeling,* McGraw Hill, New York, 1994.

Space Science--I

General Relativity and Cosmology

Total Lectures: 50 ***(Marks - 50)***

General Relativity :Minkowskispacet-time : Past and future Cauchy development, Cauchy surface. DeSitter and anti-de Sitter space-times. Robertson-Walker spaces. Spatially homogeneous space-time models. The Schwarzschild and Reissner – Nordstrom solutions. Kruskal diagram. Causal structure. Orientability. Causal curves. Causality conditions. Cauchy developments. Global hyperbolicity. The existence of Geodesics. The Causal boundary of space-time. Asymptotically simple spaces.

References:

1. *The large scale structure of space-time* - Hawking and Ellis (Camb. Univ. Press).
2. *General Relativit*y – R.M. Wald (Chicago Univ. Press).
3. *A first course in general relativity* – B.F. Schultz (Camb. Univ. Press).
4. *Gravitation and Cosmology* – S. Weinberg (J. Wiley and Sons).
5. *General Relativity, Astrophysics and Cosmology* – Raychaudhury, Banerji and Banerjee.(Springer-Verlag).

6. *General Relati*vity – M. Luduigsen (Camb. Univ. Press).
7. *Introducing Einstein's Relativity* – R d'Inverno (Clarendon Press, Oxford).

Cosmology: What is cosmology? Homogeneity and isotropy of the universe. The Weyl Postulate. The cosmological principle. General relativistic cosmological models. Cosmological observations. The Olbers Paradox. The Friedman Cosmological Models (dust and radiation models). Cosmologies with a non-zero λ . Hubble's Law, the age of the4 Universe. Gravitational red shift and Cosmological redshift. The spherically symmetric space-time: Schwarzschild solution. Patrick orbits in the Schwarzschild space-time. Newtonian approximation. Photon orbits. Birkhoffs theorem. Equilibrium of Massive spherical objects. The Schwarzschild Interior solution. The interior structure of the star. Realistic stars and gravitational collapse. White dwarfs, Neutron stars. Gravitational collapse of a homogeneous dust ball. Schwarzschild black hole. Simple idea of black hole physics.

References:

1. *General Relativity and Cosmololgy* – J.V. Narlikar.
2. *A first course in general relativity* – B.F. Schultz.
3. *Introduction to cosmology* - J.V. Narlikar.
4. *An Introduction to Mathematical Cosmology* – J.N. Islam (Camb.Univ.Press).
5. *Gravitation and Cosmology* – S. Weinberg (J. Wiley and Sons.)
6. *General Relativity, Astrophysics and Cosmology* – Raychaudhuri, Banerji and Banerjee (Springer-Verlag).
7. *Introduction to Cosmology* – M. Ross (J. Wiley and Sons).

Quantum Mechanics-I

Non Relativistic Quantum Mechanics & Relativistic Quantum Mechanics-I

Total Lectures: 50 ***(Marks – 50)***

Non Relativistic Quantum Mechanics:
Bohr's postulates of Quantum Mechanics. De Broglie's Wave. Heisenberg's principle of uncertainty. Probabilistic description. Schrödinger equation: Square well potential. One dimensional harmonic oscillator. Minimum uncertainty product. Momentum eigen functions. Box normalization. Orbital angular momentum. Hydrogen atom. Different approaches to quantum mechanics: Schrödinger representation, Heisenberg approach. Harmonic oscillator and Angular momentum as examples. Time independent peturbation theory. Spin: two component wave function. Pauli's spin matrices. Variation method. Ground state energy level of helium atom as an application of variational method. Indistinguishable particles. Exclusion principle. Multiplet structure of spectral line: exchange degeneracy. Zeeman effect- Normal and Anomalous. Pauli's equation. Density matrix. Application of density matrix for Fermions. Time dependent perturbation theory: Fremi's Golden Rule. Molecules: Hydrogen molecule: Heitler- London theory of homopolar bonding. SU(2), SU(3) matrices and their properties.

Relativistic Quantum Mechanics-I:
KLEIN-GORDON equation and its difficulties, Plane wave solutions, charge, current densities.

References:

1. L.I.Schiff: *Quantum Mechanics*(McGraw Hill)
2. E. Merzbacher: *Quantum Mechanics* (John Wiley)

3. Y.R.Waghmare: *Fundamentals of Quantum Mechanics*(Wheeler Publications)
4. J.E.House – *Fundamentals of Quantum Mechanics* (Academic Publishers)
5. R.H.Landau -- *Quantum Mechanics II*" (John Wiley)
6. L.H.Ballentine -- *Quantum Mechanics* (World Scientific)
7. P.J.E. Peebles -- *Quantum Mechanics* (Prentice Hall)

SEMESTER-IV

Paper: Math401C

Operations Research

Total Lectures: 50 ***(Marks–50)***

Introduction, Definition of O.R., Drawbacks in definition, Scope of O.R., O.R. and decision making, Application of O.R. in different sectors, Computer application in O.R. Fundamental theorem of L.P.P. along with the geometry in n-dimensional Euclidean space, hyperplane, separating and supporting plane.(5L)
Standard forms of revised simplex method, Computational procedure, Comparison of simplex method and revised simplex method, Sensitivity analysis, Bounded variable method, The Primal Dual Method. (12L)
Mathematical formulation of Assignment Problem, Optimality condition, Hungarian method, Maximization case in Assignment problem, Unbalanced Assignment problem, Restriction on Assignment, Travelling salesman problem. Standard form of Integer Programming, The concept of cutting plane, Gomory's all integer cutting plane method, Gomory's mixed integer method, Branch and Bound method. (10L)
Processing of n jobs through two machines, The Algorithm, Processing of n jobs through m machines, Processing of two jobs through m machines. (5L)
Project scheduling by PERT/CPM : Introduction, Basic differences between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM network Components and Precedence Relationships, Critical Path analysis, Probability in PERT analysis, Project Crashing, Time cost Trade-off procedure, Updating of the Project, Resource Allocation. (10L)
Deterministic Inventory control Models: Introduction, Classification of Inventories, Advantage of Carrying Inventory, Features of Inventory System, Deterministic inventory models with and without shortages. (8L)

References:

1. Wagner – *Principles of Operations Research* (PH)
2. Sasievir, Yaspan, Friedman – *Operations Research*: *Methods and Problems* (JW)
3. J. K. Sharma – *Operations Research – Theory and Applications*
4. Taha – *Operations Research*
5. Schaum's Outline Series – *Operations Research*
6. Hillie& Lieberman – *Introduction to Operations Research*
7. Swarup, Gupta &Manmohan –*Operations Research.*

OR

Paper: Math401C

Set Theory and Mathematical Logic

Total Lectures: 50 **(*Marks–50*)**

Set Theory: Axiom of choice, Zorn's Lemma, Hausdorffmaxixmality principle, well-ordering theorem and their equivalences, general Cartesian product of sets. (5L)
Cardinal numbers and their ordering, Schröder-Bernstein theorem, addition, multiplication and exponentiation of cardinal numbers, the cardinal numbers N_0 and C and their relation. (13L)
Totally orderedsets, order type, well-ordered sets, ordinal numbers, initial segments, ordering of ordinal numbers, addition and multiplication of ordinal numbers, sets of ordinal numbers, transfinite induction. (12L)
Mathematical Logic: Statement calculus: Propositional connectives, statement form, truth functions, truth tables, tautologies, contradiction, adequate sets of connectives, arguments: Proving validity rule of conditional proof,formal statement calculus, formal axiomatic theory L, deduction theorem, consequences,quantifiers, universal and existential, symbolizing everyday language.(20)

References:

1. K. Kuratowski – *Introduction to Set Theory and Topology*
2. E. Mendelson – *Introduction to Mathematical Logic*
3. R. R. Stoll – *Set Theory and Logic*
4. I. M. Copi – *Symbolic Logic*
5. W. Sierpienski– *Cardinal and Ordinal Numbers*
6. A. G. Hamilton – *Logic for Mathematicians* (Cambridge University Press).

Paper: Math402C
Group A
Graph Theory

Total Lectures: 30 ***(Marks – 30)***

Graphs, subgraphs and their various properties and characterization, complement, isomorphism, walks, paths, cycles, connected graph components, bipartite graph.

Adjacency matrix, incidence matrix.

Directed graph, adjacency matrix of a digraph.
Distance, radius and center, diameter.

Degree sequence, Havel-Hakimi theorem (statement only). (10L)

Trees, various characterizations of trees, centres of trees, spanning trees, Fundamental cycles with respect to spanning tree, Cayley's theorem on trees.
Eulerian graphs, Hamiltonian graphs, Königsberg Bridge problem.
Cut set, cut vertices, edge cut, vertex and edge connectivities.(10L)

Coloring of graphs, Chromatic number, Chromatic polynomial, edge colouring number, König theorem, Recurrence formulae.(5L)

Planar graphs, statement of Kuratowski Theorem, Eulers formula, 5 colour theorem, statement of 4 colour theorem, dual of aplanar graph.(5L)

References:

1. F. Harary – *Graph Theory* (Addison-Wesley, 1969)
2. D. West – *Introduction to Graph Theory*, PHI
3. J. A. Bondy U.S.R. Murty – *Graph Theory with Applications* (Macmillan, 1976)
4. Nar Sing Deo – *Graph Theory* (Prentice-Hall, 1974)
5. Malik, Sen, Ghosh - *Introduction to graph theory* (Cengage)
6. K. R. Pathasarthy – *Basic Graph Theory* (TMH., 1994).

Paper: MC402
Group B
Field Theory

Total Lectures:* 20** ***(Marks – 20)

Field Theory: Extension of fields, simple extensions, algebraic and transcendental extensions, splitting fields, algebraically closed fields, normal extension, separable extensions, Impossibility of some constructions by straightedge and compass, Galois field, perfect field,Galois group of automorphisms and Galois theory, solution of polynomial equations by radicals. (20L)

References:

1. Hungerford – *Algebra*, Springer
2. Patrick Morandi – *Field and Galois Theory,* Springer
3. Dummit, Foote – *Abstract Algebra,* Wiley
4. Malik, Mordeson&Sen – *Fundamentals of Abstract Algebra,* McGraw-Hill
5. I. N. Herstein – *Topics in Algebra*,Wiley
6. David A. Cox – *Galois Theory,* Wiley.

Paper: Math 403ME&Math 404ME

Advanced Differential Geometry-II

Total Lectures: 50 ***(Marks- 50)***

Affine connections, its existence, curvature, torsion of an affine connection, Riemannian manifolds, Riemannian connection,curvature tensors,sectional curvature,Schur’s theorem,geodesics in a Riemannian manifold,projective curvature tensor,conformal curvature tensor. (30L)
Submanifolds&hypersurfaces,normals, Gauss’ formulae, Weingarten equations,lines of curvature,generalized Gauss and Mainardi-Codazzi equations. (10L)
Almost complex manifolds,Nijenhuis tensor,contravariant and covariant almost analysis vector fields, F-connection. (10L)

References:

1. R. S. Mishra, *Structures on a differentiable manifold and their applications,* ChandramaPrakashan, Allahabad, 1984.
2. B. B. Sinha, *An Introduction to Modern Differential Geometry*, Kalyani Publishers, New Delhi, 1982.

3. K. Yano and M. Kon, *Structure of Manifolds*, World Scientific Publishing Co. Pvt. Ltd., 1984.
4. John M. Lee, *Introduction to Smooth Manifolds*, 2nd Ed., Springer-Verlag, 2012.
5. William H. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Academic Press, New York, 1975.
6. S. Lang, *Introduction to Differential Manifolds*, John Wiley and Sons, New York, 1962.
7. S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry*, Vol. 1, Interscience Press, Newyork, 1969.

Operator Theory and Applications-II

Total Lectures: 50 ***(Marks- 50)***

Spectral properties of bounded-linear operators in normed linear space, spectrum, regular value, resolvant of operator, closure property and boundedness property of spectrum, spectral radius. (15L)
Eigenvalues, eigen-vectors of self-adjoint operators in Hilbert space, resolvant sets, real property of spectrum of self-adjoint operators, range of spectrum, orthogonal direct sum of Hilbert space. (15L)
Spectral theorem for compact normal operators, sesquilinearfunctionals, property of boundedness and symmetry, generalisation of Cauchy-Schwarz inequality. (10L)
Unbounded operators and their adjoint in Hilbert spaces. (10L)

References:

1. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966
2. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd
3. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
4. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994
5. G.F. Simmons- *Introduction to topology and Modern Analysis* ,McGraw Hill, New York, 1963
6. N. Dunford and J.T. Schwartz-*Linear Operators, Vol-1&II*, Interscience, New York,1958
7. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3rdEdn., 1990
8. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970
9. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958
10. L.V. Kantorvich and G.P. akilov-*Functional Analysis*, Pergamon Press,1982
11. Vulikh- Functional Analysis
12. J. Tinsley Oden&Leszek F. Dernkowicz- *Functional Analysis*, CRC Press Inc, 1996.

Lipschitz-General Topology, Schaum Series.

Introduction to PDE-Constrained Optimization

Total Lectures: 50 ***(Marks- 50)***

Introduction and examples; Model control and optimization problems; Sensitivity- and adjoint-based optimization methods; First-order necessary optimality conditions; Formulation of Adjoint problems; Efficient solution strategies; SAND and NAND solution approaches; One-shot methods; Differentiate-then-discretize vs. discretize-then-differentiate approaches; Application examples: Parameter identification problems, shape optimization problems, flow control problems. (50 L)

References:

1. FrediTröltzsch– *Optimal Control of Partial Differential Equations: Theory, Methods and Applications*
2. L.T.Biegler, O.Ghattas, M. Heinkenschloss, D. Keyes, B. van Bloemen – *Real-Time PDE-Constrained Optimization,* Waanders (SIAM)
3. S. B. Hazra– *Large-Scale PDE-Constrained Optimization in Appications*(Springer).

Geometric mechanics on Riemannian Manifolds-II

Total Lectures: 50 ***(Marks- 50)***

Manifolds, tangent vectors, the differential of a map, The Lie bracket, one-forms, tensors, Riemannian manifolds, linear connections, the volume element, Laplace operators on Riemannian manifolds, gradient vector field, divergence and Laplacian, applications, Pluri-harmonic functions, uniqueness for solution of the Cauchy problem for the heat operator, the Hessian and applications, application to the heat equation with convection on compact manifolds. (10L)

Lagrangian formalism on Riemannian manifolds, a simple example, the pendulum equation, Euler–Lagrange equations on Riemannian manifolds, Laplace's equation $\Delta f = 0$, a geometrical interpretation for a Δ operator, Poisson's equation, geodesics, the natural Lagrangian on manifolds, momentum and work, force and Newton's equation, a geometrical interpretation for the potential U, harmonic maps from a Lagrangian viewpoint, introduction to harmonic maps, the energy density, harmonic maps using Lagrangian formalism, D'Alembert principle on Riemannian manifolds. (10L)

Conservation theorems, Noether's theorem, the role of killing vector fields, the energy-momentum tensor, definition of energy-momentum, Einstein tensor, field equations, divergence of the energy-momentum tensor, conservation theorems, applications of the conservation theorems, Hamiltonian formalism, momenta vector fields, Hamiltonian, Hamilton's system of equations, harmonic functions, geodesics, harmonic maps, Poincaré half-plane. (10L)

Hamilton–Jacobi Theory, Hamilton–Jacobi equation in the case of natural Lagrangian, the action function on Riemannian manifolds, Hamilton–Jacobi for conservative systems, action for an arbitrary Lagrangian, Examples, The Eiconal equation on Riemannian manifolds, applications of Eiconal equation, fundamental solution for the Laplace–Beltrami operator, fundamental singularity for the Laplacian, Laplacian momenta on a compact manifold, minimizing geodesics, minimal hypersurfaces, the Curl tensor, application to minimal hypersurfaces, Helmholtz decomposition, the non-compact case. (10L)

Radially symmetric Spaces, existence and uniqueness of geodesics, geodesic spheres, a radially non-symmetric space, the Heisenberg group, the left invariant metric, the Euler–Lagrange system, the classical action, the complex action, the volume function at the origin, mechanical curves, the areal velocity, areal velocity as an angular momentum, the circular motion, the asteroid, Noether's theorem, the first integral of energy, physical interpretation, the cycloid, solving the Euler–Lagrange system, the total energy, Galileo's law, curves that minimize a potential, the gravitational potential, minimal surfaces, the brachistochrone curve, Coloumb potential, physical interpretation, Hamiltonian approach, Hamiltonian system. (10L)

References:

1. Ovidiu Calin, Der-Chen Chang, *Geometric mechanics on Riemannian manifolds*, Springer-Verlag, 2006.

Elasticity-II

Total Lectures: 50 ***(Marks- 50)***

Vibration problems: Longitudinal vibration of thin rods, Torsional vibration of a solid circular cylinder and a solid sphere. Free Rayleigh and Love waves. (15L)
Thermoelasticity: Stress-strain relations in Thermo elasticity. Reduction of statistical thermo-elastic problem to a problem of isothermal elasticity. Basic equations in dynamic thermo elasticity. Coupling of strain and temperature fields. (25L)
Magneto-elasticity: Interaction between mechanical and magnetic field. Basic equations Linearisation of the equations. (10L)

References:

1. Y. A. Amenzade – *Theory of Elasticity* (MIR Pub.)
2. A. E. H. Love – *A treatise on the Mathematical Theory of Elasticity, CUP, 1963.*
3. I. S. Sokolnikoff– *MathematicalTheory of Elasticity, Tata McGraw Hill Co., 1977.*
4. W. Nowacki – *Thermoelasticity* (Addison Wesley)
5. Y. C. Fung- Foundations of Solid Mechanics, PHI, 1965.
6. S. Timoshenk and N. Goodies, Theory of Elasticity, McGrwa Hill Co., 1970.
7. N. I. Muskhelishvili- Some Basic Problems of the mathematical theory of Elasticity, P. Noordhoff Ltd., 1963.

Computational Fluid Dynamics

Total Lectures: 50 ***(Marks – 50)***

Introduction/motivation: What is CFD, need of CFD; Basic equations of fluid dynamics: Compressible Navier-Stokes equations, compressible Euler equations, boundary conditions; Incompressible Navier-Stokes equations, incompressible Euler equations, Stokes equations, boundary conditions; Preliminary computational techniques: Concepts of finite difference, finite volume and finite element methods, similarities & distinctions of these three methods. (5L)
Practical applications (implementations) to parabolic and elliptic equations. (5L)
Hyperbolic conservation laws: Concept of weak solution, Rankine-Hugoniot condition, Entropy condition, Entropy solution. (4L)
Discretization schemes for scalar conservation laws: Naive-scheme and Lax-Friedrichsscheme, Schemes in conservation form, consistency, stability, equivalent equation, CFLcondition, Lax-equivalence theorem. (10L)

Second order schemes, Non-smooth solutions, Lax-Wendroff theorem, Upwind and Godunov schemes. (6L)

Computation of incompressible viscous flows: Unsteady flows, staggered grid, MAC method, Higher order upwind differencing. (10L)

Steady flows: Non-linear iterations, SIMPLE formulations, implementations (10L)

References:

1. Randall J. LeVeque – Numerical Methods for Conservation Laws (Birkhäuser)
2. Kröner, D. – Numerical schemes for conservation laws (Wiley-Teubner)
3. Fletcher C. A. J. – Computational Techniques for Fluid Dynamics (Springer)
4. C. Hirsch – Numerical Computation of Internal and External Flows: The Fundamentals of Computational Fluid Dynamics (Butterworth-Heinemann)
5. P. Niyogi, S. K. Chakraborty, M. Laha – Introduction to Computational Fluid Dynamics (Pearson Education)

Space Science –II

Astrophysics

Total Lectures: 50 ***(Marks – 50)***

Application of General Relativity to Astrophysics.
Compact Objects, White dwarfs, Neutron stars and Black holes. Brief history of the formation and evolution of stars.
Schwarzschild exterior solution, Birkhoff's theorem, Schwarzschild singularity, Kruskal transformation, Schwarzschild Black hole. Motion of test particles around Schwarzschild black hole. Kerr metric and Kerr black holes (without deduction of solution). Horizons of Schwarzschild and Kerr black holes. Laws of black hole thermodynamics (statements only).
Interior of Schwarzschild metric, massive objects, Openheimer – Volkoff limit, Gravitational lensing, Quasars, Pulsars, Supernova.
Openheimer-Snydder non static dust model, Gravitational collapse.
Accretion into compact objects, Boltzmann formula, Saha Ionization equation, H-R diagram.
Plasma, black Body, Cherenkov &Synchroton Radiation.Accreation as source of radiation. Quasar as source of radition, Compton effecteffect, BremsstrallungRadition.
Formation of Galactic Structure – different Theories : Formation of our Galaxy. Formation of Galaxy in Evolutionary Universe. Formation of Galaxy in Steady State Universe. Possibility of galactic structure formation through Explosion.
Hubble's Law & Expansion of Universe – Big Bang Model. Uniformity of Large Scale Structure of the Universe. Origin of Cosmic Rays. Origin of Galaxies and the Universe.

References:

1. *The Structure of the Universe* – J.V. narlikar
2. *Astrophysics* – B. Basu
3. *Astrophysics Books*: ThanuPadmanabhan
4. *Astrophysical Concept* – M. Harmitt
5. *Galactic Structure* – A. Blauaw& M. Schmidtw
6. *Large Scale Structure of Galaxies* – W.B. Burton
7. *The Milky Way* – B.T. Bok & P.F. Bok.

8. *Cosmic Electrodynamics* – J.H. Piddington

Quantum Mechanics-II

Relativistic Quantum Mechanics-II & Quantum Field Theory

Total Lectures: 50 ***(Marks – 50)***

Relativistic Quantum Mechanics-II:

Dirac equation, Plane wave solutions, dirac matrices, charge current conservation, Spin of Dirac particle. Significance of negetive energy states, Concept of antiparticle, dirac hole theory. Non-relativistic correspondence of Dirac equation. Lorentz covariance of dirac equation. Parity, Charge conjugation, time reversal in Dirac equation.

References:

1. Schiff --*Quantum Mechanics*.
2. P.M. Mathews and K Venkatesan -- *Quantum Mechanics*.
3. P.A.M Dirac -- *The Principles of Quantum Mechanics*.

Quantum Field Theory:

Lagrangian formation, symmetries and gauge fields, Real and complex regular fields. Noether's theorem. Topology and the vaccum: the Bohm-Aharonov effect. Yang-Mills field Canonical quantization and particle interpreatation – real scalar field, complex scalar field and electromagnetic field. Path integrals and quantum mechanics. Path integral quantization. Feynman rules. Renormalization.

References:

1. L.D Landau and E.M Lifshitz-- *The Classical Theory of Fields* (Oxford, Pergamen
    a. Press).
2. L.H Ryder -- *Quantum Field Theory* (Academic Publishers).
3. C. Itzykson and j.bZuber-- *Quantum Field Theory* (McGraw Hill).
4. J.D Bjorkenand S.D Drell -- *Relativistic Quantum Fields* (McGraw Hill).
5. F. Mandi -- *Introduction to Quantum Field Theory* (Interscience, 1960).
6. R.P Feynman and A.R Hibbs -- *Quantum Mechanics and Path Integrals* (McGraw
7. Hill).

Modeling and Analysis of Biological Systems

Total Lectures: 50 ***Marks: 50***

The nature of ecosystems, Autotroph-based ecosystem, Detritus-based ecosystem, Different types of population growth, Community dynamics- succession and community responses.

Single Species Population Dynamics:

Continuous growth models – their stability analysis, Influence of random perturbations on population stability. Insect out break model- Spruce-Budworm model. General autonomous models. Delay Models

Population Dynamics of Two Interacting Species:
Introduction, Lotka-Volterra system of predator-prey interaction, Trophic function, Gauss's Model, Gause Model, Kolmogorov Model, Leslie Gower Model, Analysis of predator-prey model with limit cycle periodic behavior, parameter domains of stability. Competition models-exclusion principle and stability analysis. Models on mutualism.

Continuous models for three or more interacting species:
Three species simple and general food chain models- its stability and persistence. Models on one prey two competing predators with limited resources and living resource supporting three competing predators- stability analysis and persistence.
Reaction - Diffusion equation , Turing stability , Pattern Formation.

References:

1. Kapur, J. N., *Insight into Mathematical Modeling,* Indian National Science Academy, New Delhi, 1983.
2. Perko, L., *Differential Equations and Dynamical Systems*, Springer Verlag, 1991.
3. Kelley, W. G., Peterson, A. C., *Difference Equations- An Introduction with Applications*, Academic Press, 1991.

Paper: Math-405T(IA)

Internal Assignment (Term Paper) Math-405T(IA) is related with any topic of Mathematics and Applications and the Marks distribution is 25 Marks for written submission and 15 Marks for Seminar Presentation and 10 Marks for Viva-Voce.