

DEPARTMENT OF MATHEMATICS

BANKURA UNIVERSITY

SYLLABUS FOR RET EXAMINATION FOR ADMISSION TO Ph.D. PROGRAMME IN MATHEMATICS

Part I. Research Methodology (50% of total marks)

Part II. Subject Specific (MATHEMATICS) (50% of total marks)

Part I: Research Methodology

Meaning of Research, Objectives of Research, Motivations in Research, Types of Research, Research Approaches, Law and hypothesis. Significance of Research, Research Methods v/s Methodology, Research and Scientific Methods, Research Process, Criteria of good Research.

What is a research problem? Selecting the problem, Necessity of and techniques in defining the problem.

Purpose of the Review, Identification of the related Literature, Organizing the related Literature, Literature survey using web, handling search engines

Axiom of choice, Zorn's Lemma, Hausdorff-maximality principle, Well-ordering theorem and their equivalences, Cartesian product of sets. Relations, Equivalence relations. Cardinal numbers. Totally ordered sets, Well-ordered sets, Ordinal numbers.

Descartes' rule signs, Relations between roots and coefficients, Reciprocal equations, Binomial equations, Special roots, Cubic equations, Biquadratic equations.

Definition of Probability, Random variables, Probability Distribution Functions, Mean, Median, Mode, Skewness and Kurtosis, Binomial, Poisson, Geometric, Normal and Uniform distributions, Moment Generating Function, Characteristic Function.

Population, Sampling, Collection of data, Classification of data, Measures of Central Tendency, Measures of Dispersion, Correlation and Regression, Estimation of Parameters, Interval Estimation, Confidence Interval, Testing of Hypothesis, Null Hypothesis, Alternative Hypothesis, Types of Errors, Best Critical Region, Power of a Test.

Laplace Transform, Fourier Transform.

Linear programming problem, simplex methods, duality.

Part II: Subject Specific (Mathematics)

- **Analysis:** Countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.
- **Linear Algebra:** Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.
- **Complex Analysis:** Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.
- **Algebra:** Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in \mathbb{Z} , congruences, Chinese Remainder Theorem, Euler's ϕ -function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.
- **Topology:** basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.
- **Ordinary Differential Equations (ODEs):** Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.
- **Partial Differential Equations (PDEs):** Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.
- **Numerical Analysis:** Numerical solutions of algebraic equations, Method of iteration and

Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

- **Calculus of Variations:** Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

- **Linear Integral Equations:** Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

- **Classical Mechanics:** Generalized coordinates, Lagranges equations, Hamiltons canonical equations, Hamiltons principle and principle of least action, Two-dimensional motion of rigid bodies, Eulers dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.